



Degree of an Algebraic Expression

If an expression does not have any index or power, its index and power is regarded to be 1. Hence that is the degree of the polynomial.

For example : $5x + 2y + z$, $2x + 7y$ and $5a$ have no power on them. Therefore all the expressions have a degree of 1.

In $5a^2 + 3a$, $3k^2 + 7k$ and $5a^2$ the highest power of each expressions is 2. Therefore degree of each polynomial is 2.

In the expressions $x^3 + x$, $5a^3 + 2$, the highest power of each term is 3. So they are polynomials of third degree.

In multiplied polynomials the powers of literal factors of the expressions are added.

For example : In the polynomials (i) $3x^2y^5$, (ii) $\frac{1}{2}a^3a^2$

The degree of expression (i) is $5 + 2 = 7$

The degree of expressions (ii) is $3 + 2 = 5$.

Or we can say that (i) is a polynomial of seventh degree and (ii) is a polynomial of fifth degree.

Example : Change the following expression to algebraic expressions.

(i) The difference of 7 and a number.

(ii) The sum of a number and 5.

(iii) Product of a number and 11.

(iv) Multiply a number by 3 and add 5.

Solution : Let the number be 'a' then :

(i) $7 - a$

(ii) $a + 5$

(iii) $11a$

(iv) $3a + 5$



Addition of Algebraic Expressions

While adding an algebraic expression, we collect the like terms and add them. The sum of several like terms is the sum of the coefficients of the like terms.

An algebraic expression is a combination of numerals, literal numbers (letters) and the symbols of the fundamental operations.

Example : Add the following algebraic expressions—

(i) $5x^2y$, $-4x^2y$, $3x^2y$

(ii) $m^2 - 3mn$, $m^2 + mn$, $mn + n^2$

(iii) $6a + 8b - 5c$, $2b + c - 4a$, $a - 3b - 2c$



Solution :

$$\begin{aligned} \text{(i)} \quad & 5x^2y, -4x^2y, 3x^2y \\ & 5x^2y + (-4x^2y) + 3x^2y \text{ (monomials)} \\ & = 5x^2y - 4x^2y + 3x^2y \\ & = (5 - 4 + 3)x^2y \text{ (by collecting the coefficients of the like terms)} \\ & = 4x^2y \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & m^2 - 3mn, m^2 + mn \text{ and } mn + n^2 \\ & = (m^2 + m^2) + (-3mn + mn + mn) + n^2 \text{ (by collecting like terms)} \\ & = 2m^2 - mn + n^2 \end{aligned}$$

$$\text{(iii)} \quad 6a + 8b - 5c, 2b + c - 4a \text{ and } a - 3b - 2c$$

To solve this expression, we shall use the vertical method and write the like terms in columns

$$\begin{array}{r} 6a + 8b - 5c \\ - 4a + 2b + c \\ + a - 3b - 2c \\ \hline 3a + 7b - 6c \\ \hline = 3a + 7b - 6c \end{array} \quad \text{(trinomials)}$$



Subtraction of Algebraic Expressions

Two add two or more algebraic expressions, we collect the like terms from each and add them. While subtracting, we write the given algebraic expressions in two rows in such a way that the like terms occur one below the other. The expression from which we have to subtract is in the first row and the expression which is to be subtracted is in the second row.

Example :

- (i) $7x^2y^2$ from $-11x^2y^2$
- (ii) $11ab^2 + ab$ from $16ab^2 - 3ab$
- (iii) $4a + 5b - 3c$ from $6a - 3b + c$

Solution :

$$\begin{aligned} \text{(i)} \quad & 7x^2y^2 \text{ from } -11x^2y^2 \\ & (-11x^2y^2) - (7x^2y^2) \quad \text{(monomials)} \\ & = -11x^2y^2 - 7x^2y^2 \\ & = -17x^2y^2 \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & 11ab^2 + ab \text{ from } 16ab^2 - 3ab \\ & = (16ab^2 - 3ab) - (11ab^2 + ab) \quad \text{(binomials)} \\ & = 16ab^2 - 11ab^2 - ab \\ & = (16ab^2 - 11ab^2) + (-3ab - ab) \\ & = 5ab^2 - 4ab \end{aligned}$$

$$\text{(iii)} \quad 4a + 5b - 3c \text{ from } 6a - 3b + c$$

We shall use the vertical method to solve this algebraic expression. We will write like terms below each other and change the signs of the terms to be subtracted.





$$\begin{array}{r}
 6a - 3b + c \quad (\text{trinomials}) \\
 - (4a + 5b - 3c) \quad (\text{by changing the signs}) \\
 \hline
 = 2a - 8b + 4c
 \end{array}
 \qquad
 \begin{array}{r}
 6a - 3b + c \\
 - (4a + 5b - 3c) \\
 \hline
 2a - 8b + 4c
 \end{array}$$

Example :

From the sum of $5a + 6b - 7c$ and $9a - 16b - 2c$ subtract $7a - 10b + 9c$.

Solution:

Adding the two expressions we get,

$$\begin{array}{r}
 5a + 6b - 7c \\
 + 9a - 16b - 2c \\
 \hline
 14a - 10b - 9c
 \end{array}$$

Now by subtracting $7a - 10b + 9c$ from the above sum we get,

$$\begin{array}{r}
 14a - 10b - 9c \\
 - (7a - 10b + 9c) \\
 \hline
 = 7a - 18c
 \end{array}
 \qquad
 \text{or}
 \qquad
 \begin{array}{r}
 14a - 10b - 9c \\
 - 7a + 10b - 9c \\
 \hline
 7a + 0 - 18c
 \end{array}$$



Exercise 5.1

1. Change the statements to algebraic expressions.

- Subtract 7 from a number
- 3 added to a number multiplied with 5.
- The sum of number and 11 multiplied by 7.
- 2 subtracted from the product of 6 and a number
- 7 multiplied to the difference of 8 and a number.

2. Find the degree of the following polynomials.

- $\frac{3}{5}$
- $4a + 3b + c$
- $3x^2y^3 - 5x^2yz^2 - 3x^3$
- $5x^6 - y^3 + 7xy^8$
- $5x^4 - 3x^2 + 4x + 5$

3. Add the following :

- $2x - 5y - 4z$ and $3x + 5y - 4z$
- $-3a^2 - 2b^2 + 4$, $-a^2 - 5b^2 - 8$ and $2a^2 + 4b^2 + 6$
- $2x^2 - 3x + 6$, $x - 3x^2$, $-4 + x^2$, $-2x + 7x^2 - 3$
- $2x^2 + 4y^2 + 2$, $-2x^2 - 5y^2 + 7$, $-x^2 + y^2$
- $3mn + 4pq + 2$, $-3pq - 4mn - 1$, $-pq - mn$

4. Subtract the following :

- $(2a - b)$ from $(5a + 2b - 3)$
- $(-m^2 - 2n)$ from $(6m^2 - 3n + 8)$
- $(-5p + q + 7s)$ from $(7p - 8q + r)$





(d) $(-4x^2 + 5y^2 - xy + 9)$ from $(-x^2 - 4y^2 + xy - 7)$

(e) $(4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$ from $(18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q)$

8. Find the perimeter of triangle whose sides are $m+n$, $-2m+n$ and $4m+3n$, respectively, in terms of m and n .
9. Subtract $-2x^2 + y^2 - xy + x$ from the sum of $x^2 + 4y^2 - 6xy$, $x^2 - y^2 + 2xy$, $y^2 + 6$ and $x^2 - 4xy$.



Multiplication of Algebraic Expressions

Laws Of Signs For Multiplication

- (i) The product of two numbers having like signs is positive.
- (ii) The product of two numbers having unlike signs is negative.

If x and y are two variables, then

$$\begin{aligned} (+x)(+y) &= (+xy) \\ (-x)(-y) &= (+xy) \\ (+x)(-y) &= (-xy) \\ (-x)(+y) &= (-xy) \end{aligned}$$

Rule Of Multiplication : While multiplying two or more variables, the powers of like variables are added.

For example : $x^2 \times x^3 = x^{2+3} = x^5$



Multiplication of A Monomial by A Monomial

Rule : Product of two monomials
= (Product of their numerical coefficients) $\times$ (Product of their variables)

Example :

If $a - \frac{1}{a} = 3$, find the value of $a^2 + \frac{1}{a^2}$

Solution:

we have, $a - \frac{1}{a} = 3$

Squaring both sides, we get $\left(a - \frac{1}{a}\right)^2 = 3^2 \Rightarrow a^2 + \frac{1}{a^2} - 2 \times a \times \frac{1}{a} = 9$

$$\Rightarrow a^2 + \frac{1}{a^2} - 2 = 9$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 9 + 2 = 11$$

Example :

If $a + \frac{1}{a} = 8$. Find the value of $a^2 + \frac{1}{a^2}$

Solution:

we have, $a + \frac{1}{a} = 8$.

Squaring both sides. we get $\left(a + \frac{1}{a}\right)^2 = 8^2$

$$\Rightarrow a^2 + \frac{1}{a^2} + 2 \times a \times \frac{1}{a} = 64$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 64 - 2 = 62$$





Multiplication of A Polynomial by A Monomial

Rule : To multiply a polynomial by a monomial, we commonly use distributive law.

$$a \times (b+c) = a \times b + a \times c$$

Example :

Find the product of :

$$6a^2b^2 \times (2a^2 - 4ab + 5b^2)$$

Solution:

$$\begin{aligned} & 6a^2b^2 \times (2a^2 - 4ab + 5b^2) \\ &= (6a^2b^2 \times 2a^2) + [6a^2b^2 \times (-4ab)] + (6a^2b^2 \times 5b^2) \\ &= 12a^4b^2 - 24a^3b^3 + 30a^2b^4 \end{aligned}$$

Example :

Simplify: $a(a^2 + a + 1) + 5$ and find its value for $a = -1$

Solution:

$$a(a^2 + a + 1) + 5 = a^3 + a^2 + a + 5$$

Substituting, $a = -1$, we have

$$\begin{aligned} & (-1)^3 + (-1)^2 + (-1) + 5 \\ &= -1 + 1 - 1 + 5 \\ &= 4 \end{aligned}$$

Thus, the value of $a(a^2 + a + 1) + 5$ for $a = -1$ is 4.

Example :

Subtract : $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

Solution:

$$\begin{aligned} \text{We have, } & 3l(l - 4m + 5n) = 3l^2 - 12lm + 15ln \\ \text{and, } & 4l(10n - 3m + 2l) = 40ln - 12lm + 8l^2 \end{aligned}$$

On subtraction, we have,

$$\begin{array}{r} 40ln - 12lm + 8l^2 \\ + \quad 15ln - 12lm + 3l^2 \\ \hline (-) \quad (+) \quad (-) \\ \hline 25ln \quad + \quad 0 \quad + 5l^2 \end{array}$$

$$\begin{aligned} \text{Thus, } \{4l(10n - 3m + 2l)\} - \{3l(l - 4m + 5n)\} &= 25ln + 5l^2 \\ &= 5l(5n + l) \end{aligned}$$



Multiplication of A Binomial by A Binomial

Rule : Let us consider two binomials $(a+b)$ and $(c+d)$, respectively. Using distributive law of multiplication over addition, we have

$$\begin{aligned} (a+b) \times (c+d) &= a \times (c+d) + b \times (c+d) \\ &= (axc + axd) + (bxc + bxd) \\ &= ac + ad + bc + bd \end{aligned}$$

Example :

Multiply: $(2pq + 3q^2)$ and $(3pq + 2q^2)$

Solution:

$$\begin{aligned} & (2pq + 3q^2) \times (3pq + 2q^2) \\ &= 2pq(3pq + 2q^2) + 3q^2(3pq + 2q^2) \\ &= \{(2pq) \times (3pq) + (2pq) \times (2q^2)\} + \{(3q^2) \times (3pq) + (3q^2) \times (2q^2)\} \\ &= 6p^2q^2 + 4pq^3 + 9pq^3 + 6q^4 \\ &= 6p^2q^2 + 13pq^3 + 6q^4 \end{aligned}$$





Multiplication of A Binomial by A Trinomial or Multiplication of A Polynomial by A Polynomial



Golden Rule; Product of two monomials = (product of their numerical co-efficients) x (product of their variables) known method of (Horizontal method)

By using the distributive law of multiplication over addition twice, we may find the product of two binomials

$$\begin{aligned}(a+b) \times (c+d) &= a \times (c+d) + b \times (c+d) \\ &= (a \times c) + (a \times d) + (b \times c) + (b \times d) \\ &= ac + ad + bc + bd\end{aligned}$$



Exercise 5.2

1. Find the product of following monomials :

- (a) $5a^2b$ and $-2ab^2$ (b) $-6x^2y$ and $-4x^2y^2$
(c) $-4xy^2$ and $-2x^3y$ (d) $-3xy$ and $7x^3y^3$

2. Obtain the volume of rectangular boxes with the following length, breadth and height, respectively :

- (a) $2xy, 3x^2y, 4xy^2$ (b) $2p, 6q, 7r$
(c) $3x, 4xy, 2xyz$ (d) a, a^2, a^3

3. Simplify the expressions and evaluate them as directed :

- (a) $x(x-3)+2$, for $x=1$
(b) $3y(2y-7)-3(y-4)-63$, for $y=-2$

4. Find the area of a rectangle whose sides are $5x$ and $2y$.

5. Find the area of a square whose sides are $4a^2$.

6. Solve :

- (a) $\{5x(3-x)\} + \{6x^2-13x\}$ (b) $\{2ab(a+b)\} - \{3ab(a-b)\}$

7. Find the product of :

- (a) $(a^2) \times (2a^{22}) \times (4a^{26})$ (b) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$
(c) $\left(\frac{-10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$ (d) $x \times x^2 \times x^3 \times x^4$

8. Simplify :

- (a) $(a+b) (a^2-ab+b^2)$
(b) $(2x^2-5x+4) \times (x^2+7x-8)$
(c) $(3a^2+b^2) \times (2a^2+3b^2)$
(d) $(2m^3-5m^2-m+7) \times (3-2m+4m^2)$





Division of Algebraic Expressions

An algebraic expression of the form.

$$a + bx + cx^2 + dx^3 + ex^4 + \dots$$

When, $a, b, c, d, e, \dots$ are the constants and x is a variable and is known as polynomial in x .

(i) The product of two numbers having like signs is positive.

(ii) The product of two numbers having unlike signs is negative.

Rule of Division : If x is a variable and a and b are positive integers, such that $a > b$, then

$$x^a \div x^b = x^{a-b}$$



Division of A Monomial by A Monomial

Rule : Quotient of two monomials

= (Quotient of their numerical coefficients) $\times$ (Quotient of their variables)

Solved Example :

Divide :

(i) $15x^2y^5$ by $-3xy$

(ii) $-12x^3y^2z$ by $-2xyz$

Solution:

(i) $\frac{15x^2y^5}{-3xy} = \left(\frac{15}{-3}\right)x^{(2-1)}y^{(5-1)} = -5xy^4$

(ii) $\frac{-12x^3y^2z}{-2xyz} = \left(\frac{-12}{-2}\right)x^{(3-1)}y^{(2-1)}z^{(1-1)} = 6x^2y^1z^0 = 6x^2y$



Division of A Polynomial by A Monomial

Golden Rule and method:- Quotient of two monomials = (Quotient of their numerical co-efficients) $\times$ (Quotient of their variables).

i.e; $\frac{15x^2y^5}{-3xy} = \left(\frac{15}{-3}\right)x^{(2-1)}y^{(5-1)} = -5xy^4$

Rule : To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Solved Example :

Divide :

$9x^6 + 24x^5 - 18x^3$ by $3x^2$

Solution:

$(9x^6 + 24x^5 - 18x^3) \div 3x^2$

$= \frac{9x^6}{3x^2} + \frac{24x^5}{3x^2} - \frac{18x^3}{3x^2}$

$= 3x^4 + 8x^3 - 6x$





Division of A Polynomial by A Polynomial



To divide a polynomial by an another polynomial, follow the steps given below:

Step-1 : Arrange the terms of the divisor and dividend in descending order of their degrees.

Step-2 : Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

Step-3 : Multiply all the terms of the divisor by the first term of the quotient and then subtract the result from the dividend.

Step-4 : The remainder obtained (if any), becomes the new dividend.

Step-5 : Repeat the above process, until you get the remainder either as 0 or a polynomial of degree less than that of a divisor.

Example :

Divide :

$$(x^2 - 4x + 4) \text{ by } (x - 2)$$

Solution:

$$\begin{array}{r} x-2 \overline{) x^2 - 4x + 4} \quad (x-2 \\ \underline{x^2 - 2x} \\ -2x + 4 \\ \underline{-2x + 4} \\ 0 \end{array}$$

$$\therefore \text{Quotient} = x - 2, \quad \text{Remainder} = 0$$

Example :

Divide :

$$(2x^3 + x^2 - 5x - 2) \text{ by } (2x + 3)$$

Solution:

$$\begin{array}{r} 2x+3 \overline{) 2x^3 + x^2 - 5x - 2} \quad (x^2 - x - 1 \\ \underline{2x^3 + 3x^2} \\ -2x^2 - 5x \\ \underline{-2x^2 - 3x} \\ -2x - 2 \\ \underline{-2x - 3} \\ 1 \end{array}$$

$$\therefore \text{Quotient} = x^2 - x - 1, \quad \text{Remainder} = 1$$



Exercise 5.3

1. Divide :

(a) $45x^2y^3$ by $5xy$

(b) $32abc^3$ by $-4ac$

(c) $-64a^2b^2c$ by $-8abc$

(d) $-24xyz^2$ by $12xyz$

2. Divide :

(a) $4x^4 - 12x^2 + 36x$ by $4x$

(b) $6x^2y - 8xy + 10xy^2$ by $-2xy$

(c) $9a^2b^2 - 12ab^2 + 15a^2b^3$ by $3ab$

(d) $21a^4 + 14a^3 - 7a^2$ by $-7a^2$

3. Divide the following and write the quotient and remainder :

(a) $(2x^2 + 3x + 1)$ by $(x + 1)$

(b) $5a^3 - 15a^2 + 12a + 3$ by $(3 - 3a + a^2)$

(c) $(x^4 + 4x^3 - 2x^2 + 10x - 25)$ by $(x + 5)$

(d) $(6m^2 - 31m + 47)$ by $(2m - 5)$

$$\begin{aligned}(x+a)(x+b) &= x(x+b) + a(x+b) && \{ \text{Using distributive law} \} \\ &= x^2 + xb + ax + ab \\ &= x^2 + (a+b)x + ab\end{aligned}$$

$$\text{Thus, } (x+a)(x+b) = x^2 + (a+b)x + ab$$



Points to Remember :

- Expressions that contain one, two or three terms are called monomials, binomials and trinomials, respectively. In general, any expression with three or more terms can be called a polynomial.
- Terms with same literal factors are called like terms. If the literal factors are not the same, they are called unlike terms.
- Only like terms can be added or subtracted.
- When a monomial is multiplied by another monomial, the product is a monomial.
- While multiplying a polynomial by a monomial, every term in the polynomial is to be multiplied by the monomial.
- While multiplying two polynomials, every term in one polynomial is to be multiplied by every term in the other polynomial.
- While dividing a polynomial by a monomial, every term of the polynomial is to be divided by the monomial.
- To divide a polynomial by a polynomial, the long division method is used.
- On performing long division, we arrange the dividend and the divisor in the standard form, that is, arrange them in descending order of the divisor.
- On performing long division, if the remainder is 0, the divisor is a factor of the dividend.
- The degree of the remainder has to be less than the degree of the divisor.
- An identity is an equation which is true for all values of the variables that it contains.
- The important and useful identities are:
 Identity 1: $(a + b)^2 = a^2 + 2ab + b^2$
 Identity 2: $(a - b)^2 = a^2 - 2ab + b^2$
 Identity 3: $(a + b)(a - b) = a^2 - b^2$ (It is called the difference of two squares.)
 Identity 4: $(x + a)(x + b) = x^2 + (a + b)x + ab$





EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options.

- (a) $4x^2y + 3x^3$ is a—
 (i) Monomial ☐ (ii) Binomial ☐ (iii) Trinomial ☐ (iv) None of these ☐
- (b) If x and y are two variables, then $(-x) \times (-y) = ?$
 (i) xy ☐ (ii) $(-x)(y)$ ☐ (iii) $x \times (-y)$ ☐ (iv) None of these ☐
- (c) The area of a square of side $6x$ is—
 (i) $6x$ ☐ (ii) $12x^2$ ☐ (iii) $36x^2$ ☐ (iv) $36x$ ☐
- (d) $x^a \div x^b = ?$
 (i) x^{a-b} ☐ (ii) x^{b-a} ☐ (iii) x^{a+b} ☐ (iv) None of these ☐
- (e) $(a-b)^2 = ?$
 (i) $(a^2 - 2ab + b^2)$ ☐ (ii) $(a^2 + 2ab + b^2)$ ☐ (iii) $(a^2 + b^2)$ ☐ (iv) $(a^2 - b^2)$ ☐
- (f) The quotient when $-56xyz^3$ divided by $8xyz$ is—
 (i) $-7z^2$ ☐ (ii) $7z^2$ ☐ (iii) $8xyz$ ☐ (iv) $-56z^3$ ☐
2. The age of father is $13xy - 6x^2 + 4a^2 - 1$. The age of the son is $25x^2 + 16xy - 3b^2 - 2$. Find the difference of their ages.
3. The perimeter of a triangle is $6a^2 - 4a + 9$ and two of its sides are $a^2 - 2a + 1$ and $3a^2 - 5a + 3$. Find the length of the third side of the triangle.
4. Add $3x^2 + 4x - 2$ to the product of $(3x - 4)$ and $(x + 5)$.
5. From the product of $(xy + y + 1)$ and $(y - 6)$ subtract $4xy^2 + 9y^2$.
6. Add the product of $(4ab + b)$ and $(b - 7)$ to the product of $(-3ab + 1)$ and $(b + 2)$.
7. What should be subtracted from $14a^2 + 13a - 15$ to make it divisible by $7a - 4$?
8. What should be subtracted from $4x^3 + 8x^2 + 8x + 7$ to make it divisible by $2x^2 - x + 1$?
9. The perimeter of a triangle is $7x^2 - 17xy + 5y^2 + 8$ and two of its sides are $4x^2 - 7xy + 4y^2 - 3$ and $5 + 6y^2 - 8xy + x^2$. Find the third side of the triangle.
10. The expression $2x^4 - x^3 - 3x^2 + 5x - 2$ should be divided by which expression to get $x^2 + x - 1$ as the quotient?



HOTS

1. If $a^2 + \frac{1}{a^2} = 27$, find the value of $a^4 + \frac{1}{a^4}$
2. If $x = 11$, find the value of
 (a) $x^2 + 2$ (b) $x^4 + 4$



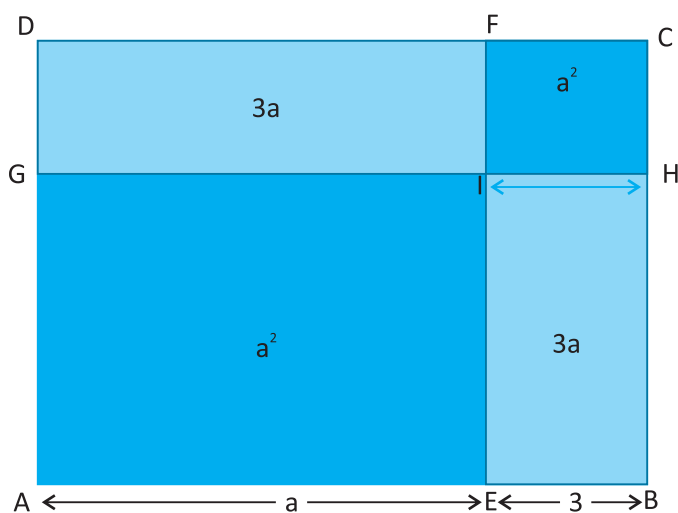
Lab Activity

Objective : To visualise and reinforce the concept of identities.

Materials Required : Chart paper / graph paper and colored sketch pens.

Procedure : To prove $(a + 3)^2 = a^2 + 6a + 9$ using a diagram proceed with the following steps:

Step 1 : Draw a square ABCD and divide it into two squares and two rectangles as shown using a black tram.



Step 2 : Let AE measure a units and EB measure 3 units since $AB = AE + EB$, we have $AB = (a + 3)$ units.

Step 3 : Shade the two squares in red colour and two rectangles in green colour to distinguish between square and rectangles.

Area of square ABCD = Area of square AEIG + Area of rectangle EBHI + Area of square IHCF + Area of rectangle DGIF

$$(a + 3)^2 = a^2 + 3a + 3^2 + 3a$$

$$(a + 3)^2 = a^2 + 6a + 9 \text{ sq. units.}$$

We can put any numerical value of a to verify the equation.

